h, or enthalpy, is a measure of the total energy in a fluid. This includes energy due to the random motion of its molecules (temperature), and its pressure. A rocket converts the high enthalpy of the stagnant combustion chamber gas into a lower enthalpy value, plus directed kinetic energy. However, the sum of these two must equal the original enthalpy of the gas in the chamber. That is, the amount of enthalpy you have in the chamber is all there is, none is created as the gas expands in the nozzle, it is just converted. This is quantified as:

$$h_0 = h_e + \frac{v_e^2}{2}$$

Where “e” means the value at the nozzle exit, and “0” is the value in the combustion chamber. The equation can be converted, by using a well known simplification for enthalpy, called the calorically perfect gas assumption. This approximation states that enthalpy is the product of the gas temperature, and its heat capacity, Cp.

$$C_p T_0 = C_p T_e + \frac{v_e^2}{2}$$

$$T_0 = T_e + \frac{v_e^2}{2 C_p}$$

$$C_p \left(1 - \frac{T_e}{T_0}\right) = \frac{v_e^2}{2 T_0}$$

From there, the specific heat, Cp, can be written in terms of γ, the adiabatic index, which is just a property of the gas, the universal gas constant (Ru), a value which is the same for any gas, and the molecular weight, MW, of the gas.

$$\frac{\gamma}{\gamma - 1} \frac{R_u}{MW} \left(1 - \frac{T_e}{T_0}\right) = \frac{v_e^2}{2 T_0}$$

We then solve for exit velocity:

$$v_e = \sqrt{\frac{2\gamma}{\gamma - 1} \frac{R_u T_0}{MW} \left(1 - \frac{T_e}{T_0}\right)}$$

The ratio between chamber temperature, T0, and nozzle exit temperature, Te, is rewritten as a pressure ratio, with an exponent. Finally giving us the exit velocity as:

$$v_e = \sqrt{\left(\frac{2\gamma R_u}{\gamma - 1}\right)\left(\frac{T_0}{MW}\right)\left(1 - \left(\frac{P_e}{P_0}\right)^{\frac{\gamma - 1}{\gamma}}\right)}$$

It is clear that specific impulse depends on:

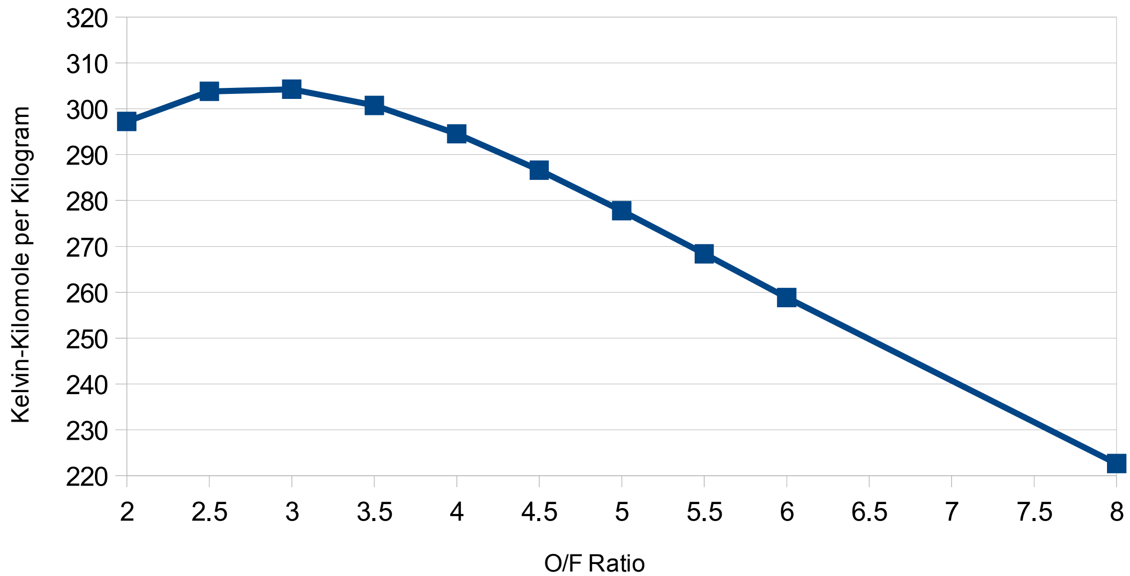
1. MW: The molecular weight of the gas
2. γ : The adiabatic index
3. T0: Chamber temperature
4. Pe/P0: Nozzle pressure ratio

Nozzle pressure ratio can be controlled independently of combustion chemistry, just make a bigger nozzle.

The adiabatic index, γ, does not vary greatly for the problem we are considering. For steam, it is approximately 1.3, versus 1.4 for pure hydrogen.

Therefore, the only parameter strongly dependent on chemistry is the ratio between chamber temperature and molecular weight.

This relationship is graphed below for hydrogen-oxygen combustion.



In short, the peak for this ratio occurs nowhere near the stoichiometric O/F of 8. Chamber temperature and molecular weight are equally important in the final specific impulse. However, chamber temperature is only somewhat affected by mixture ratio, whereas molecular weight is strongly affected.

Sacrificing a small amount of chamber temperature to get a much lower molecular weight still gives a net gain in specific impulse.

Note: There have been several simplifying assumptions made. The actual peak to the above curve will be somewhat further right, but again, nowhere near 8.