

Faster-Than-Light Space Warps, Status and Next Steps

Eric W. Davis *

Institute for Advanced Studies at Austin and Tau Zero Foundation, Austin, Texas, 78759

Implementation of faster-than-light (FTL) interstellar travel via traversable wormholes or warp drives requires the engineering of spacetime into very specialized local geometries. The analysis of these via Einstein's General Theory of Relativity demonstrates that such geometries require the use of "exotic" matter. One can appeal to quantum field theory to find both natural and phenomenological sources of exotic matter. Such quantum fields are disturbed by the curved spacetime geometry they produce, so their energy-momentum tensor can be used to probe the back-reaction of the field effects upon the dynamics of the FTL spacetime, which has implications for the construction of FTL spacetimes. Related issues are the problem of turning space warps on/off. Also, the production, detection and deployment of natural exotic quantum fields are seen to be key technical challenges in which basic first steps can be taken to experimentally probe their properties. FTL spacetimes also possess features that challenge the notions of momentum conservation and causality. The status of these important issues is addressed in this report, and recommended next steps for further investigations are identified in an effort to clear up a number of technical uncertainties in order to progress the present state-of-the-art in FTL spacetime physics.

Nomenclature

a	= starship coordinate acceleration
α, β	= 0...3, tensor indices denoting spacetime coordinates
c	= speed of light
D	= distance of stellar destination from Earth
$d\ell$	= element of proper distance
dt	= element of coordinate time
$d\tau$	= element of proper time
d^4x	= (3+1)-dimensional spacetime volume element
Δ	= warp bubble wall thickness
$\Delta\ell$	= proper distance of travel as measured by space travelers
Δt	= travel time through traversable wormhole as measured by outside remote static observers
$\Delta\tau$	= proper time of travel as measured by space travelers going through traversable wormhole
E_{warp}	= total negative energy required to construct warp bubble
E_{wh}	= total negative energy required to construct traversable wormhole
G	= universal gravitational constant
$G_{\mu\nu}$	= Einstein curvature tensor
g_{\oplus}	= acceleration of gravity at Earth's surface
$g_{\mu\nu}$	= spacetime metric tensor
\hbar	= Planck's reduced constant
\mathcal{L}	= Lagrangian density
ℓ_{C}	= local radius of curvature of FTL space warp
ℓ_{P}	= Planck length
M_{ship}	= starship mass
M_{warp}	= absolute value of equivalent mass of total negative energy threading FTL space warp

* Senior Research Physicist, 11855 Research Blvd., AIAA Associate Fellow.

μ, ν	=	0...3, tensor indices denoting spacetime coordinates
Ω	=	(3+1)-dimensional spacetime volume
p_i	=	spatial pressure/tension/stress components of matter or quantum field
$\phi(r)$	=	redshift function defining proper time lapse through traversable wormhole
$\hat{\phi}$	=	quantum field operator
$R (> 0)$	=	radius of warp bubble
r	=	radial (distance) coordinate
r_{throat}	=	suitable measure of linear dimension (width or diameter) of traversable wormhole throat
r_w	=	radial distance from center of M_{warp}
ρ_E	=	energy density of matter or quantum field
ρ_{vp}	=	negative vacuum polarization energy density
σ	=	inverse of warp bubble wall thickness
$T_{\mu\nu}$	=	energy-momentum-stress tensor
$\hat{T}_{\mu\nu}(x)$	=	quantum energy-momentum-stress operator
$\langle \hat{T}_{\mu\nu} \rangle$	=	quantum expectation value of energy-momentum-stress operator
t	=	x^0 , time coordinate
τ	=	proper time or total elapsed proper time
T	=	observed time interval or total elapsed coordinate time
v	=	$v(r)$, radial velocity through traversable wormhole throat
v_{warp}	=	dimensionless speed of warp bubble, [$v_{\text{warp}}: (0, \infty)$]
$x^1 \dots x^3$	=	space coordinates
$ 0_{\text{in}}\rangle$	=	initial quantum vacuum state

I. Introduction

Spacecraft propulsion physics is based on Newtonian laws of mechanics, which are dependent upon the expenditure of propellant kinetic energy or the beaming of propulsive energy to induce thrust-generating momentum transfer. Investigators have proposed interstellar propulsion modes based on a large variety of nuclear rockets, electric drives, beamed energy propulsion, sails, ramjets, etc. The limiting speed of space flight, independent of any propulsion mode, is the speed of light. For interstellar rendezvous missions, traditional Newtonian rocket propulsion suffers from enormous mass ratios for spacecraft cruise velocities $> 0.05c$, if we are to constrain the travel time to within 100 years for a one-way voyage. If we increase the cruise velocity to relativistic or even ultra-relativistic speed, then the mass ratio becomes worse due to the relativistic Lorentz factor. This shows that Newtonian and relativistic rockets will consist mostly of propellant in order to propel the propellant plus a miniscule payload. R. H. Frisbee¹ performed a comprehensive technical evaluation of several different interstellar propulsion technologies and concluded that the most likely candidates for fast ($\sim 0.5c$) interstellar missions suffer from either a need for extraordinary amounts of propellant, or a need for very large system infrastructure, or a host of significant feasibility issues. He concluded that there is no single concept that is without potentially significant shortcomings.

At non-relativistic cruise speeds, it will take explorers tens of thousands to hundreds of thousands of years to reach stellar destinations. While at near-relativistic or even ultra-relativistic cruise speed, mission rendezvous times will range from several days to several tens of years depending on the total flight distance and cruise speed. And the associated special relativistic time dilation results in decades to thousands of years of elapsed time on Earth. This is an undesirable outcome for any interstellar mission.

A proposed alternative is to dispense with Newtonian or relativistic propulsion and instead deploy a “space warp” to travel faster-than-light (FTL) to the distant stars. The primary general requirements for putative FTL space warps should be:

1. The rocket equation no longer applies.
2. The starship travel time via the FTL space warp should take ≤ 1 year as measured by the starship passengers and outside remote static observers.
3. Proper time as measured by starship passengers should not be dilated by relativistic effects.

4. The FTL space warp-induced tidal-gravity accelerations acting between different parts of the starship passengers' body should be $\leq 1 g_{\oplus}$ inside of the FTL space warp.
5. The speed of the starship while inside the FTL space warp should be $< c$.
6. The starship (made of ordinary matter) must not couple strongly to the material that generates the FTL space warp.
7. The FTL space warp should not have an event horizon.
8. There should be no singularity of infinitely collapsed matter residing inside or outside of the FTL space warp.

There are two types of FTL space warps that satisfy all of these requirements: traversable wormholes and warp drives.² There are transient, static and dynamical traversable wormhole spacetime geometries while warp drives are dynamical spacetime geometries. FTL space warps are a class of exact solutions to Einstein's General Theory of Relativity (GR). GR simply expresses the following classical spacetime physics of gravitation and matter: a local distribution (source) of matter tells spacetime how to curve while curved spacetime tells a source of matter how to move.

Spacetime curvature is encoded by the Einstein curvature tensor $G_{\mu\nu}$ which is a function of the spacetime metric tensor $g_{\mu\nu}$ and its derivatives, and the energy-momentum tensor $T_{\mu\nu}(x)$ encodes the density and flux of the energy and momentum contained in a source of matter located at position $x \equiv x^{\mu}$ in spacetime. $g_{\mu\nu}$ encodes the geometry and topology of spacetime and is itself a function of the spacetime coordinates x^{μ} . Tensors are matrix quantities and the compact form of the GR field equations are mathematically expressed as $G_{\mu\nu} = (8\pi G/c^4)T_{\mu\nu}$ (MKS units are assumed throughout). This represents a coupled set of nonlinear partial differential equations with an associated set of mathematical identities and symmetry conditions. It is important to note that in contrast to other classical field theories, the field $g_{\mu\nu}$ plays a dual role as both the quantity which describes the dynamical aspects of spacetime/gravity and the quantity which describes the background spacetime structure.

In the list above, item 1 implies that FTL space warps might be considered as a class of "space drives".³ That is because GR replaced gravity (as a fundamental force of nature) with curved spacetime geometry, i.e., curved spacetime is gravitation and vice versa. Item 5 requires that space travelers inside a FTL space warp not violate Einstein's Special Theory of Relativity (SR) by always remaining inside their local light cone along the trajectory between departure and arrival points, whereas the FTL space warp acts to tilt their local light cone over thus giving rise to apparent FTL motion as seen by outside static observers whose local light cones are unaffected by the FTL space warp. Items 2 and 3 ensure that the travelers' local clock rate is the same as the local clock rate of outside static observers.

Implementation of FTL space warps generally requires the engineering of spacetime into very specialized local geometries as dictated by the combined requirements of items 1 through 8. The analysis of these via GR plus the resultant matter equations of state demonstrate that such geometries require the use of "exotic" matter fields. Quantum field theory provides the source of both natural and phenomenological exotic matter with examples of the former having been experimentally demonstrated in the laboratory. In general, such quantum fields are gravitationally disturbed by the local background spacetime geometry they produce, so their energy-momentum tensor can be used to probe the back-reaction of quantum field effects upon the dynamics of the FTL space warp itself. In regions of spacetime curvature, vacuum polarization effects can also lead to an important phenomenon that may provide a useful feedback mechanism on the formation of the FTL space warp. These two issues have implications on the construction of FTL space warps. A related issue is the problem of turning FTL space warps on/off. Also, the production, detection and deployment of natural exotic quantum fields are seen to be key technical challenges in which basic first steps can be taken to experimentally probe their properties. FTL spacetimes also possess features that challenge the notions of momentum conservation and causality. K. F. Long⁴ provides an excellent overview of these technical issues. The present technical status of these important issues will be discussed in this report, and recommended next steps for further investigations are identified in an effort to clear up a number of technical uncertainties in order to progress the present state-of-the-art in FTL spacetime physics.

II. Exotic Matter and the Classical Hawking-Ellis Energy Conditions

When studying FTL spacetime physics, the appropriate strategy is to decide beforehand on a definition of the FTL space warp that one desires and then decide what the spacetime geometry should be. Given the desired

geometry, use the general relativistic field equations to calculate the distribution of matter required to produce this geometry. Then one needs to assess whether the required distribution of matter is physically reasonable and whether it violates any basic rules of physics, etc. In the case of traversable wormhole and warp drive spacetimes, it turns out that they and their variants universally violate some or all of the classical Hawking-Ellis energy conditions.⁵ Matter that violates these conditions is called “exotic.” (See the discussion in Reference 2 for further details.) However, the requirement that the energy conditions must be obeyed by all forms of matter in nature is spurious because generic spacetime geometries and quantum field theory violate some or all of the energy conditions.²

Quantum field theory has the remarkable property of allowing states of matter containing local regions of negative energy density or negative fluxes,⁶ so these are the primary sources of exotic matter for producing FTL space warps. The different types of matter that have negative energy density/fluxes are described in Reference 2. Consideration of the predominantly quantum sources of exotic matter and their associated FTL space warps require that they be analyzed within the well established paradigm of semi-classical quantum gravity theory. In this paradigm, the curvature of spacetime (or gravitation) $G_{\mu\nu}$ is treated as a classical field that is associated with a given quantum state of the source matter, $\langle \hat{T}_{\mu\nu} \rangle$. $\langle \hat{T}_{\mu\nu} \rangle$ denotes the quantum expectation value of the energy-momentum operator $\hat{T}_{\mu\nu}(x)$ in the given quantum state. Thus, semi-classical quantum GR is compactly expressed as

$$G_{\mu\nu} = (8\pi G/c^4) \langle \hat{T}_{\mu\nu} \rangle. \quad (1)$$

For the case of quantum exotic matter that is of relevance to FTL space warp studies, the quantum state to be taken in the expectation value is predominantly the ground, or vacuum, state because negative energy is a *sub-vacuum* energy that derives from distorted vacuum states. Note that in postulating Eq. (1), the superposition principle for quantum matter states is lost because different matter states are associated with different spacetimes. Another consequence of Eq. (1) is what happens to the gravitational dynamics when one makes a measurement of the location of a source of quantum matter. If Eq. (1) continues to hold after the quantum state and location of the matter are resolved by measurement, then the gravitational field must change in a discontinuous, acausal manner. This exhibits the nonlocality associated with $\langle \hat{T}_{\mu\nu} \rangle$ in the physical (3+1)-dimensional[†] quantum field theory, which is a property of quantum back-reaction. $\langle \hat{T}_{\mu\nu} \rangle$ acts as the source of gravity in Eq. (1) in addition to describing part of the physical structure of the quantum (matter) field at the point x . It therefore plays an important part in any attempt to model a self-consistent dynamics involving the gravitational field coupled to the quantum field. It is this so-called *back-reaction* of the quantum processes on the background spacetime geometry – gravitational dynamics modified by the gravitationally induced $\langle \hat{T}_{\mu\nu} \rangle$ – that is of primary concern to FTL space warp designers. The consequences of this to the assembly and turning on/off of FTL space warps will be discussed in more detail later.

A. FTL Space Warp Energetics

We know how to make small quantities of negative energy in the lab via the Casimir effect and squeezed vacuum states.² However, we do not know if it is possible to make the much larger quantities of total negative energy required to produce traversable wormholes (E_{wh}) and warp drives (E_{warp}):²

$$E_{\text{wh}} = -10^{44} r_{\text{throat}} \quad (2a)$$

$$E_{\text{warp}} = -10^{44} v_{\text{warp}}^2 R^2 \sigma, \quad (2b)$$

where the numerical factors in Eqs. (2a) and (2b) come from c^4/G . The gravitational coupling of the finite starship mass with the warp bubble leads to the condition that the net total energy stored in the warp bubble should be less than the total rest-energy of the starship itself, which places a strong constraint upon the (dimensionless) speed of the warp bubble:²

[†] In GR and quantum field theory, the “dimensional spacetime” notation $(n+1)$ is commonly used, where $n = 1, 2, 3$ is the number of space dimensions and 1 is the time dimension.

$$v_{\text{warp}} \leq \left[\left(7.41 \times 10^{-28} \right) \left(\frac{M_{\text{ship}} \Delta}{R^2} \right) \right]^{\frac{1}{2}}. \quad (3)$$

Equation (3) is due to the intrinsic nonlinearity of the general relativistic field equations.

FTL space warp energy requirements and the development by M. Visser and collaborators of new energy conditions and constraints that replaced the Hawking-Ellis energy conditions are discussed in Reference 2. The new energy conditions led to the remarkable result that arbitrarily small amounts of negative energy are required to construct traversable wormholes. But there are a few non-fatal consequences of this. First, the smaller the amount of negative energy density (or general exotic matter) used in wormholes, the closer they are to becoming vacuum Schwarzschild wormholes which are not traversable. Second, as the amount of negative energy density decreases, the longer it will take a starship to traverse the wormhole as measured by the clocks of outside static observers. Third, as the amount of negative energy density decreases, the wormhole becomes more prone to destabilization by even minute amounts of infalling (positive) matter because this matter will be enormously blueshifted by the time it reaches the throat. However, P. K. F. Kuhfittig^{7,8} derived wormhole solutions that satisfy the traversability constraints and require arbitrarily small amounts of negative energy density. C. Barcelo and M. Visser^{9,10} proposed a possible solution by using classical non-minimally coupled scalar fields as sources of exotic matter for wormhole maintenance. In the case of relatively weak scalar fields and coupling constants, one can get large sustainable, albeit temporary, classical negative energy fluxes even in flat spacetime.

B. Present Status of the Quantum Inequalities

The Quantum Inequality (QI) conjecture constrains the magnitude and duration of negative energy densities (associated with a free quantum field) relative to those of an underlying reference vacuum state, i.e., the QIs place bounds on quantum violations of the classical energy conditions and the second law of thermodynamics.^{11,12} Technically speaking, the QIs give a mathematical bound on averages of the quantum expectation value of a free field's energy-momentum tensor in the vacuum state, where the QIs are averaged along an observer's timelike or null worldline using ad hoc time sampling functions. Moreover, the QIs dictate that the more negative the energy density is in some time interval, the shorter the duration of the interval such that an inertial observer cannot see arbitrarily large negative energy densities that last for arbitrarily long time intervals – an inertial observer must encounter compensating positive energy density no later than a time T , which is inversely proportional to the magnitude of the initial negative energy density. This contrasts with the classical Hawking-Ellis energy conditions, which imply that an inertial observer who initially encounters some negative energy density must encounter compensating positive energy density at some arbitrary time in the future. The QIs were devised to place strong bounds on the production of negative energy density and their corresponding violation of the second law of thermodynamics, and thus constrain the construction of FTL spacetimes.

The QIs have been proven using arbitrary smooth and compactly-supported time sampling functions. And they have been proven for free fields, e.g., the electromagnetic field, the Dirac field in both (1+1)- and (3+1)-dimensional (flat) Minkowski spacetime, the massless and massive minimally coupled scalar fields, massive spin-1, and Rarita-Schwinger (spin-3/2 fermion) fields. They have also been proven in lower dimensional toy spacetimes.

Spatial- and spacetime-averaged QIs are required to bring them into correspondence with the new energy conditions that replaced the Hawking-Ellis energy conditions for FTL spacetimes.^{13,14-16} No purely spatial QIs exist in (3+1)-dimensional spacetime even though they exist in (2+1)-dimensional spacetime, and none exist along null worldlines in (3+1)-dimensional spacetime even though they exist in (2+1)-dimensional spacetime. Although there is weak evidence for the existence of spacetime-averaged QIs in (3+1)-dimensional spacetime, no explicit analytical formulas can be derived. The reason for this is that the techniques used to prove worldline QIs cannot be generalized to the spacetime-averaged case, because of the nonlocality component of quantum back-reaction in the physical (3+1)-dimensional spacetime.

Simple models of quantum states involving negative energy densities, e.g., squeezed vacuum states of a massive scalar field, can exhibit a subtle intertwining of negative energy density with positive energy density. But it is uncertain how generic this behavior is. A. Borde, L. H. Ford and T. A. Roman¹⁷ addressed this by showing that one can constrain large classes of possible spatial distributions of negative energy density in flat spacetime using simply the Averaged Weak Energy Condition[†] and the QIs. This approach may be complementary to spacetime-averaged

[†] Weak Energy Condition (WEC): $\rho_E \geq 0$, $\rho_E + p_i \geq 0$. The Averaged WEC is defined by the proper time integral of the energy-momentum tensor over timelike geodesics.

QIs. They recast the QI conjecture into a new program which seeks to study the allowed spatial distributions of negative energy density in quantum field theory. Their study models free massless scalar fields in (1+1)-dimensional Minkowski spacetime. Several explicit examples of spacetime-averaged QI were evaluated to allow or rule out some particular model (spatial) distributions of negative energy density. Their analysis showed that some geometric configurations of negative energy density can either be ruled out or else constrained by the QI restrictions placed upon the allowable spatial distributions of negative energy density. And they found allowable negative energy density distributions in which observers would never encounter the accompanying positive energy density distribution so long as the QI restrictions and corresponding classical Hawking-Ellis energy conditions are violated. The extent to which these results can be generalized to analytic formulas in (3+1)-dimensional curved spacetime, with or without boundaries and interacting fields, will likely remain intractable because of the effects of nonlocality in quantum back-reaction.

Deriving QIs for interacting quantum fields in flat or curved spacetimes is technically daunting. This is a key issue because interacting fields could turn out to be far more important than free fields for the production of large amounts of negative energy distributed over large spatial regions. K. D. Olum and N. Graham¹⁸ recently constructed a (2+1)-dimensional spacetime model of two interacting scalar fields in which the energy density can be negative and static in certain regions. It is not yet known whether this arrangement can be made negative and static over an arbitrarily large spatial region. From theory and experiment, we know that large static negative energy densities associated with vacuum states are concentrated in narrow spatial regions, e. g., inside a Casimir cavity or in the region near the Schwarzschild radius in the Boulware vacuum (exterior to a black hole) where the energy density is everywhere negative as seen by static observers.¹⁹ This is also found to be true in the case of time-domain squeezed vacuum states where very small negative energy density pulses occur between the alternating larger pulses of positive energy density.¹² But there are other forms of squeezed vacuum states where the energy density is negative, but not necessarily static, everywhere as seen by static observers.

L. H. Ford and N. F. Svaiter^{20,21} showed that quantum vacuum fluctuations can be focused by a cylindrical parabolic mirror whereby the sign of the vacuum energy density can be made negative at the focal line of the mirror. The smallest relevant length scale in their model is not the proper distance to the mirror, which can be arbitrarily far away, but instead the distance to the focal point of the mirror, which is much smaller. If this is correct, then the prescription that the flat spacetime QIs should also hold for sampling times small compared to the smallest proper radius of spacetime curvature or the smallest proper distance to a boundary would need to be refined. It is important to confirm whether there might be additional relevant geometrical length scales to consider when applying the flat spacetime QIs to curved spacetimes or to spacetimes with boundaries, other than just the smallest proper radius of curvature or the smallest proper distance to a boundary.

Cosmological inflationary expansion models that involve negative energy densities should be examined as possible examples of a field that does not obey the QIs. This also applies to the Boulware vacuum external to black holes. In particular, M. Visser²² points out that observational astrophysical data indicate that large amounts of “exotic matter” are required to exist in the universe in order to account for the observed cosmological evolution parameters. This implies widespread cosmological violations of the QIs in addition to the widespread violations of the classical Hawking-Ellis energy conditions. Also, the historical analysis of the QIs does not take into account the negative energy densities associated with energy-momentum tensor fluctuations, which could be important for areas such as cosmological inflation.²³⁻²⁵

L. H. Ford and T. A. Roman²⁶ showed that the QI bounds imply that either a wormhole must have a throat size no larger than a few thousand ℓ_P or (typically) the negative energy must be confined to an extremely thin band around the throat. A similar problem afflicts warp drive spacetimes.²⁷⁻²⁹ S. V. Krasnikov^{29,30} argues to the contrary by constructing an explicit counterexample for generalized FTL spacetimes showing that the relevant QI breaks down even in the simplest FTL space warp cases. Since classical fields would not be subject to the QIs, C. Barcelo and M. Visser^{9,10} proposed classical non-minimally coupled scalar fields as sources of exotic matter for wormhole maintenance. But are such classical Hawking-Ellis energy condition-violating classical fields physically real? One answer to this question is to consider that the conformally coupled scalar field is deemed to be physically reasonable because it faithfully mimics certain behaviors of the electromagnetic field. On the other hand, P. K. F. Kuhfittig^{7,8} constructed a wide class of traversable wormhole solutions that are claimed to satisfy the QI.

It is clear from these considerations that the efficacy, consistency and reliability of the QIs remain in doubt with no clear resolution forthcoming. It should be noted that the QIs have not been experimentally tested across the broad class of natural sources of negative energy density while observational astrophysics appears to disprove them.

III. FTL Space Warp Assembly

GR does not include assembly instructions for FTL space warps. How does one deploy, shape and control negative energy density or other forms of exotic matter in order to assemble a traversable wormhole or a warp drive? Does one need to pull a traversable wormhole from out of the putative quantum spacetime foam (multiply-connected spacetime structure) and enlarge it to macroscopic scale, or will wormhole engineers need to produce extremely large spacetime curvatures to “punch a hole through space” (the change-in-topology-of-spacetime problem), and then stabilize either approach using negative energy density? The spacetime-geometrical instructions for constructing a traversable wormhole are even more vague: 1) take two copies of flat spacetime, one each near the departure and destination stars; 2) remove identical hypervolumes[§] from each spacetime; 3) identify at the boundaries (i.e., “sew” the two leftover “holes” together to form a throat).³¹ Warp drives entail the turning-on of the local inflation/contraction of, or rolling through, a nearly-flat spacetime. Such processes appear to be difficult to analyze theoretically.

The good news is that the problem of changing the topology of spacetime has been solved. Studies of a new quantum gravity approach in 2-dimensional space and (2+1)-dimensional spacetime, which is not a theory of gravitons but of shape-shifting spaces, explicitly show that the topology of space can change, thus supporting the creation of traversable wormholes.³² M. Visser³¹ provides a comprehensive analysis of the physics, construction, and stability for a number of different classes of traversable wormholes. There is no single such reference available for the varieties of warp drives.

Because “matter tells spacetime how to curve” in GR, the focus of assembly analysis should simply be on addressing how to produce and deploy the large amount of negative energy density required to assemble, stabilize and maintain FTL space warps. In the case of traversable wormholes, M. S. Morris, K. S. Thorne and U. Yurtsever³³ suggested using the negative energy density of the Casimir vacuum inside a cavity comprised of two identical, perfectly conducting concentric spherical plates with equal electric charges. The electric charges on the two plates produce a repulsive electrostatic force that counterbalances the attractive Casimir force acting on the plates. The plates’ radius is 1 AU (= mean Earth-Sun distance) which is taken to be the characteristic size of the wormhole throat. We showed that a 1 AU wormhole throat requires a plate separation of 1.57×10^{-12} m (35% smaller than the electron’s Compton wavelength) to produce a Casimir vacuum energy density of -2.16×10^{20} J/m³.² There is no technology known today that can implement such minuscule cavity plate separations with plate dimensions on the order of 1 AU. In addition, such minuscule plate separations are unrealistic because the Casimir effect switches over to the non-retarded field behavior of van der Waals forces when plate separations go below the wavelength (≈ 15 nm) where the plates are no longer perfectly conducting.² No corresponding model has been proposed for constructing warp drives.

We proposed two conceptual techniques for producing and controlling negative energy density via the quantum optically squeezed (electromagnetic) vacuum states produced by nonlinear degenerate parametric amplifier crystals and photonic crystal waveguides.² These concepts produce small amounts of negative energy density and it is not yet known whether they can be scaled up to produce very large amounts of negative energy density. The negative energy density associated with a gravitationally squeezed vacuum (a.k.a. gravitational vacuum polarization) was also evaluated and its magnitude was shown to be extremely minute.² However, gravitational vacuum polarization energy appears to provide positive feedback on FTL space warps against quantum back-reaction, which will be discussed later.

L. H. Ford and N. F. Svaiter^{20,21} proposed a cylindrical parabolic mirror that focuses quantum vacuum fluctuations and produces negative vacuum energy at the focal line of the mirror. Perhaps a carefully placed array of such mirrors could produce an intense region of large-magnitude negative energy density, which could be shaped and controlled by reorienting one or more elements of the array. One could imagine a warp drive starship using an array of externally mounted Ford-Svaiter mirrors to produce, shape and orient a bubble of negative energy that in turn produces the equivalent warp bubble spacetime around the starship. In the case of traversable wormholes, perhaps an array of Ford-Svaiter mirrors are deployed to open and stabilize a traversable wormhole throat followed by a small fleet of Ford-Svaiter mirror spacecraft which enter the throat to extend it out to the target destination. The technical details will have to be worked out.

The technical issue of capturing and storing negative energy density is not considered because free-space sources or projectors of negative energy density appear to be a more desirable option for producing FTL spacetimes than stored negative energy density, and because there is very little technical literature that addresses how to capture and store negative energy (see, e.g., Reference 12). This technical issue requires future theoretical investigation.

[§] A hypervolume is a 3-dimensional or (2+1)-dimensional surface of (3+1)-dimensional spacetime.

A. Alternative Warp Drive Approaches

What are the alternatives for constructing a warp drive, and can the requirement for astronomically large amounts of negative energy density be dramatically reduced via new approaches? Alcubierre's warp drive required on the order of a negative (equivalent) galactic mass to implement, so C. V. D. Broeck³⁴ modified the warp bubble geometry and reduced the energy requirement to on the order of a few negative (equivalent) solar masses. S. V. Krasnikov²⁹ achieved a similar result by specifying a "warp tube" spacetime in which a starship traveling one way at ultra-relativistic speed creates a tube-shaped space warp behind itself, and then it would return by traveling back through the warp tube at FTL speed. A Krasnikov starship can return from its interstellar journey shortly after it left no matter how far away it traveled. This is a highly convoluted implementation. However, these alternative warp drive implementations in GR cannot overcome the key constraint that warp speeds will remain absurdly low ($\ll c$) no matter how low the amount of negative energy one requires.²

A small number of new warp drive approaches using recent quantum gravity theories have introduced thought provoking bypasses around this problem. H. G. White³⁵ and H. G. White and E. W. Davis³⁶ put the Alcubierre warp drive metric into its canonical form using Rindler's method so that Alcubierre's warp drive could be extended to extra-space dimensional brane-world theory and reinterpreted as a spacetime expansion boost, or scalar multiplier, acting on the starship initial velocity instead of Alcubierre's expansion/contraction of spacetime via the York Time. This extension to extra-space dimensional brane-world theory recasts the Alcubierre warp drive's negative energy requirement into the cosmological dark energy equation of state, which becomes the exotic matter source for the White-Davis brane-world warp drive. H. G. White^{**} recently optimized the warp bubble wall thickness (thicker wall \rightarrow lower negative energy density) and its corresponding brane-world metric, and discovered the possibility for high-frequency pulsing of the warp bubble, all of which results in a tremendous reduction of the total negative energy required to produce a warp bubble. His calculations show that the total integrated negative energy required to produce a 10-m diameter warp bubble at $v_{\text{warp}} = 10c$ is on the order of the negative rest-mass-energy of the Voyager 1 interplanetary flyby probe. If this model holds up to further scrutiny, then it represents a potential game changer for warp drives.

R. K. Obousy and G. Cleaver³⁷ and R. K. Obousy and A. Saharian³⁸ propose another alternative to Alcubierre's warp drive which exploits the negative Casimir vacuum energy of the compactified extra-space dimensions in brane-world theories. They propose that the vacuum energy of Einstein's cosmological constant is a function of the size of the extra-space dimensions, and that if one could locally control the size of extra-space dimensions, and thus locally control the cosmological constant, then one could facilitate the local expansion/contraction of spacetime surrounding a starship. Their proposal applied the results from brane-world theories which show that the Casimir vacuum energy of extra-space dimensions is related to the cosmological constant (a.k.a. dark energy). Their model estimated that the total negative energy required to produce a very fast ($\gg c$) warp drive is on the order of the negative (equivalent) mass of Jupiter. Their model also predicts a universal upper limit to warp speed of $10^{32}c$ in contrast to Eq. (3). No technological implementations for these alternative warp drives have been proposed yet. Also, an extension of the new energy conditions and the QIs to brane-world quantum gravity warp drives has not been performed, so it is not yet clear whether these energy constraints will have any impact or relevance in this case.

It is not yet known whether alternative gravity theories (e.g., Jordan-Brans-Dicke scalar-tensor, scalar-vector-tensor, higher curvature Gauss-Bonnet, Dilatonic Einstein-Gauss-Bonnet, Einstein-Cartan, etc.) allow warp drive solutions. However, both alternative gravity and quantum gravity theories allow traversable wormhole solutions. These alternative theories remain speculations until one of them is universally adopted to supercede GR and quantum field theory. For this reason, alternative gravity theory FTL space warps will not be considered further.

IV. The Effects of Quantum Back-Reaction

Within the context of assembling and turning FTL space warps on/off, an important issue to consider is the computation of the back-reaction of quantum vacuum fluctuations of the source quantum matter field upon the curved spacetime geometry (i.e., on gravitation). Here, one considers a quantum field in a classical curved background that is similar to a driven harmonic oscillator. In this case, the field $\hat{\phi}$ is analogous to a quantum oscillator with infinitely many degrees of freedom. The metric $g_{\mu\nu}$ plays the role of the classical gravitational background (or an external field) that drives the oscillator. The main task is to calculate the expectation value of the

^{**} Private communication, NASA-Johnson Space Center, Houston, TX, 2012.

energy-momentum operator $\langle 0_{in} | \hat{T}_{\mu\nu}(\hat{\phi}, g_{\alpha\beta}) | 0_{in} \rangle \equiv \langle \hat{T}_{\mu\nu}(\hat{\phi}, g_{\alpha\beta}) \rangle_{vac}$ assuming an initial vacuum state $|0_{in}\rangle$ for $\hat{\phi}$. In general, $\langle \hat{T}_{\mu\nu}(\hat{\phi}, g_{\alpha\beta}) \rangle_{vac}$ describes the effects of the quantum back-reaction.

A consequence of back-reaction is that quanta (or particles, e.g., Hawking radiation) are produced by an external gravitational field. In the environment of an external gravitational field (or curved spacetime geometry), the vacuum fluctuations of $\hat{\phi}$ are not only *excited* but also *deformed* by the external gravitational (or any other) field. This deformation is the shift of zero-point energy of the “field harmonic oscillators” in the classical curved background, and it is called the *vacuum polarization*. There is no unique way to separate the “particles” and the vacuum polarization contributions in the total energy-momentum operator. The number density of the particles produced by the gravitational field depends on the whole preceding history of the evolution of the field. Therefore, the contribution of the produced particles to the energy-momentum operator is described by non-local expressions – the produced particles are quantum entangled such that the gravitational field responds in a discontinuous, acausal manner. On the other hand, the vacuum polarization is related to the “deformation of the vacuum fluctuations” by the gravitational field at a given moment of time and hence it is described by local terms that depend only on the local curvature characterizing the gravitational field at a given location. Because the notion of a particle in an external gravitational field is not well-defined in semi-classical quantum gravity, one cannot unambiguously split the local and nonlocal contributions to the induced $\langle \hat{T}_{\mu\nu}(\hat{\phi}, g_{\alpha\beta}) \rangle_{vac}$. The leading local contributions to the induced $\langle \hat{T}_{\mu\nu}(\hat{\phi}, g_{\alpha\beta}) \rangle_{vac}$ can be calculated for an arbitrary curved background, while the determination of the nonlocal contributions is a much more difficult problem that has not been solved in the general case. It is important to strongly emphasize that the quantum nonlocality of $\langle \hat{T}_{\mu\nu}(\hat{\phi}, g_{\alpha\beta}) \rangle_{vac}$ is a property that does not exist in any toy (1+1)- or (2+1)-dimensional quantum field theory; it is a property that is unique only to quantum field theory in our physical (3+1)-dimensional spacetime. Taking into account the back-reaction of quantum fields, the general relativistic field equations are simply given by Eq. (1) whereby the induced $\langle \hat{T}_{\mu\nu}(\hat{\phi}, g_{\alpha\beta}) \rangle_{vac}$ simultaneously accounts for the produced particles and for the vacuum polarization effects (see, e.g., Reference 39 for the technical details). In addition, a cloud of vacuum polarization has negative energy vacuum stress that violates the WEC.

W. A. Hiscock,⁴⁰ S. Finazzi, S. Liberati and C. Barcelo,⁴¹ and C. Barcelo, S. Finazzi and S. Liberati⁴² evaluated a modified Alcubierre warp drive in a toy (1+1)-dimensional spacetime and concluded that it is unstable against back-reaction which produces the following effects: 1) intense thermal Hawking radiation floods the warp bubble thus endangering the starship crew and destabilizing the bubble; 2) vacuum polarization can counteract the destabilizing thermal Hawking radiation but the system doesn’t get ahead at superluminal speed; 3) a future event horizon forms in front (past event horizon forms aft) of the starship, thus causally disconnecting the interior of the warp bubble from its forward edge; the starship bridge on the inside cannot steer the bubble or turn it on/off, and information cannot be sent from the starship to the region where the negative energy field needs to be controlled; and 4) each of the previous problems go away at sublight speed and the warp drive becomes stable and controllable. No analysis has been done to reproduce these back-reaction effects in physical (3+1)-dimensional spacetime, and it may not be possible to do the calculations due to the technicalities to be discussed below. The claims that these toy model analyses prove the impossibility of warp drives are premature and technically unsupported.

The study of quantum back-reaction is still nascent, and so in the foreseeable future it will not be possible to say anything definitive about its full effects on the feasibility of FTL space warps in physical (3+1)-dimensional spacetime. There’s also the problem of not being able to identify a unique vacuum or other quantum field state for $\langle \hat{T}_{\mu\nu}(\hat{\phi}, g_{\alpha\beta}) \rangle$ in an external gravitational field. Related to this is the large renormalization freedom^{††} in the very definition of $\langle \hat{T}_{\mu\nu}(\hat{\phi}, g_{\alpha\beta}) \rangle$; the coefficients in front of the local curvature terms are not fixed and even if they were, it is not certain whether Eq. (1) would still make sense.

The situation as it stands now is that all theoretical results and interpretations for warp drives in toy (1+1)-dimensional spacetime models are simply not relevant to our physical (3+1)-dimensional spacetime. Only in (1+1)-dimensional spacetime does the conformal anomaly essentially give the result for $\langle \hat{T}_{\mu\nu}(\hat{\phi}, g_{\alpha\beta}) \rangle$, and most of the

^{††} Renormalization refers to the large class of non-unique mathematical procedures used to remove the infinite zero-point energy from $\langle \hat{T}_{\mu\nu}(\hat{\phi}, g_{\alpha\beta}) \rangle$ in order to obtain a finite result.

intuition from (1+1)-dimensional spacetime does not carry over to our physical (3+1)-dimensional spacetime. For instance, in (1+1)-dimensional spacetime $\langle \hat{T}_{\mu\nu}(\hat{\phi}, g_{\alpha\beta}) \rangle$ is local, in the sense that a change of spacetime geometry at a given location will induce a corresponding change in $\langle \hat{T}_{\mu\nu}(\hat{\phi}, g_{\alpha\beta}) \rangle$ at the same location. However, this is not true in our physical (3+1)-dimensional spacetime, which is the essence of the nonlocality in quantum field theory effects. In short, in (3+1)-dimensional spacetime you must do the mode sums and this is almost always impossible to do. These conclusions are also true for the case of traversable wormholes, so a separate discussion on the effects of quantum back-reaction on traversable wormholes is not required.

One last important note on quantum back-reaction is the potential for the negative energy vacuum stress of vacuum polarization to act as a positive feedback on the assembly, turning-on/off and maintenance of warp drives and traversable wormholes in (3+1)-dimensional spacetime. Recall that vacuum polarization is a local back-reaction effect that depends on the local radius of curvature characterizing the spacetime curvature at a given location. Thus, there will be vacuum polarization collocated with the spacetime curvature associated with the warp bubble and the traversable wormhole throat. Correspondingly, the negative vacuum polarization energy intermingles with the negative energy that threads these two FTL space warps. This has the effect of increasing the amount of negative energy threading FTL space warps, thus counterbalancing the effects of any putative sources of instability. An estimate for the amount of negative vacuum polarization energy density that contributes positive feedback to the FTL space warp is:²

$$\rho_{vp} \approx -\frac{2\pi^2 \hbar c}{\ell_C^4} \approx -\frac{2\pi^2 \hbar G^2 M_{\text{warp}}^2}{c^3 r_w^6} \approx -(3.43 \times 10^{-79}) \frac{M_{\text{warp}}^2}{r_w^6}. \quad (4)$$

It is of interest to point out that the first expression on the right for ρ_{vp} in Eq. (4) is characteristic of expressions for the Casimir vacuum energy density.²

The r_w^{-6} dependence in Eq. (4) indicates that the bulk of the negative vacuum polarization energy will be concentrated in the immediate vicinity of the FTL space warp, and the numerical coefficient indicates that the magnitude of this energy will be extremely minute even for a M_{warp} typical of wormholes and warp drives. Equation (4) implies that vacuum polarization will not provide much of a positive feedback to the FTL space warp unless M_{warp} is extremely large and/or r_w is extremely small. However, Eq. (4) is not entirely correct because it assumed the ℓ_C of a spherical body of mass M_{warp} . A more accurate derivation requires inserting the ℓ_C of a specific type of FTL space warp. ℓ_C is defined by the inverse square root of the typical Riemann curvature tensor component in a local orthonormal frame, and so it will need to be calculated for the case of a typical traversable wormhole and a typical warp drive in order to obtain accurate estimates for ρ_{vp} in both cases. Equation (4) also serves as a crude estimate for the magnitude of the nonlocal contributions (e.g., particle production and discontinuous acausal effects upon the FTL spacetime curvature) to the induced back-reaction. This is an extremely negligible effect in FTL space warps which is consistent with the fact that they lack event horizons, whereas the magnitude of both local and nonlocal back-reaction effects are usually quite large in the case of black holes.

V. Detecting Negative Energy for FTL Space Warp Construction

If one had the means to technologically produce negative energy density via a source of quantum matter (in a sub-vacuum state), how would they be able to detect or observe it so that they could control it? A scheme for detecting negative energy in experiments was proposed by P. C. W. Davies and A. C. Ottewill⁴³ who studied the response of switched particle detectors to static negative energy densities and negative energy fluxes. Their model is based on a free (massless) scalar field in (3+1)-dimensional (flat) Minkowski spacetime and utilized a simple generalization of the standard monopole detector, which is switched on and off to concentrate the measurements on periods of isolated negative energy density (or negative energy flux). Their model included an explicit switching factor whereby five different switching functions (based on data windowing theory) are defined and evaluated. Their results shed light on the response of matter (detectors) to pulses of negative energy of finite duration, and they showed that negative energy should have the effect of enhancing deexcitation (i.e., induce cooling) of the detector. They also concluded that the enhanced cooling effect they discovered cannot be used to draw a thermodynamic conclusion because their modeling was restricted to first order in perturbation theory. It is not possible at first order to determine whether the enhanced cooling effects are due to the small violation of energy conservation expected in any process in which a general quantum state collapses to an energy eigenstate, or whether they predict a systematic

reduction in the energy of the detector which has serious thermodynamic implications. Their results are model dependent and there is not always a simple relationship between the strength of the negative energy density/flux and the behavior of the monopole detector.

On the other hand, squeezed states of light, which are *darker than vacuum*, have regions with sub-vacuum quantum (electromagnetic) fluctuations, i.e., regions with less vacuum fluctuations than undisturbed vacuum.^{2,12} The energy density of any sub-vacuum region is negative. R. E. Slusher and collaborators^{44,45} and A. L. Robinson^{46,47} were the first to experimentally observe these sub-vacuum regions in squeezed light. Numerous other experiments followed, which employed variations on the experimental devices and techniques used to generate squeezed light and measure its sub-vacuum fluctuation pulses. Those early experimental devices later gave way to the development and use of balanced homodyne detectors (BHDs).

For example, K. Schneider et al.⁴⁸ describe their compact and efficient source of amplitude-squeezed light. Their experiment used a semi-monolithic degenerate MgO:LiNbO₃ optical parametric amplifier pumped by a frequency-doubled Nd:YAG laser at 532 nm. They employed injection-seeding of the amplifier by a 1064 nm wave to provide active stabilization of the cavity length and stable operation. At a pump power of 380 mW, their device detected a maximum noise reduction of 6.5 dB in the amplitude fluctuations of the 0.2 mW 1064 nm wave, while the average detected noise reduction in continuous operation over 14 minutes was 6.2 dB. They reported a squeezing of 7.2 dB in the emitted wave.

H. Hansen et al.⁴⁹ describe their experimental time-domain BHD device. They developed a pulsed BHD for precise measurement of the electric field quadratures of pulsed optical quantum states. A high level of common mode suppression (> 85 dB) and low electronic noise (730 electrons per pulse) in their device provides a signal-to-noise ratio of 14 dB for measurement of the quantum noise of individual pulses. Their device achieved a signal-to-noise ratio of 14 dB at a pulse repetition rate of up to 1 MHz, enabling high-accuracy quantum measurements to be carried out in a short time. They performed a quantum tomography of the coherent state as a test for their device, and the Wigner function and density matrix were reconstructed with 99.5% fidelity while their detector exhibited 91% quantum efficiency. Their detection system can also be used for ultrasensitive balanced detection in continuous wave mode.

What has not been experimentally measured yet are the sub-vacuum fluctuations and their corresponding negative energy density inside a Casimir cavity. The negative energy density inside a Casimir cavity is static, so this is not a time or frequency domain situation. P. Marecki^{50,51} proposed a modified BHD that can be used to quantify the fluctuations of the quantum electric field and the associated spectral density for the ground state of the quantum electric field in Casimir cavity geometries, and he predicted a position- and frequency-dependent pattern of BHD responses if a device of this type is placed inside a cavity. The proposed BHD allows for the direct detection of sub-vacuum fluctuations and provides a spatial-frequency mapping of the negative energy density inside the cavity. This offers a potential new characterization of ground states in Casimir cavity geometries, which would provide an understanding of the negative energy densities present in some regions in these geometries, which addresses the Davies-Ottewill negative energy detector hypothesis.

Marecki's proposed BHD-Casimir cavity experiment is an excellent incremental first step toward addressing the problem of detecting negative energy density.

VI. Conservation of Momentum in FTL Space Warps

We examined the problem of whether FTL space warps obey the conservation of momentum and postulated a simple scenario for how momentum conservation might be upheld in the case of a generic traversable wormhole and in the case of a generic warp drive.² Upon further study, we discovered that the approach taken may need to be revised because there are far more important technical subtleties involved that were overlooked, which are as follows.

A long-standing result of mathematical physics is Noether's theorem⁵² which shows that continuous symmetries in physical laws lead to the existence of conserved quantities. Simply put, the invariance of the action integral, or Hamilton's principle of least action, under continuous Lorentz transformations

$$\delta A = \delta \int_{\Omega} \mathcal{L} d^4x = \delta \int_{\Omega'} \mathcal{L}' d^4x' = 0 \quad (5)$$

leads to the appearance of certain conservation laws for the classical or quantum fields associated with the Lagrangian density, \mathcal{L} . Equation (5) is known as Noether's theorem. In Eq. (5), δ symbolizes the variation of the action integral A and the prime denotes Lorentz-transformed quantities. The result of having physical laws that don't

change with time is that the total energy must be conserved. Similarly, momentum conservation follows mathematically from the fact that physical laws don't change when you move from one location in spacetime to another. In GR, it looks like energy-momentum conservation has the force of mathematical proof, but the tensor calculus used in GR satisfies its defining symmetries automatically, via the tensor notation $T^{\mu\nu}_{;\nu} = 0$ (the covariant divergence of a source of matter), where the semicolon denotes covariant differentiation of $T^{\mu\nu}$.

As stressed by Einstein in GR, a requirement of the spacetime language is that the solutions of the field equations should be regular. In other words, the field solutions should not only be continuous but also analytic – continuously differentiable to all orders and without any singularities – everywhere. This is based on the empirical requirement that the (local) flat spacetime limit of the general field theory in a curved spacetime must include the laws of conservation – of energy, linear momentum, and angular momentum. According to Noether's theorem, the analyticity of the field solutions is a necessary and sufficient condition for the existence of these conservation laws. Strictly speaking, there are no conservation laws in GR because (covariantly) a time rate-of-change of some function of the spacetime coordinates in a curved spacetime cannot be separated from the rest of the formulation that can go to zero. Consequently, the laws of conservation apply strictly only to the local flat space domain. The conservation laws are then a local limit of global laws in GR. In the general relativistic global field laws, a time rate-of-change can no longer be separated, by itself, from a (3+1)-dimensional differential change of functions mapped in a curved spacetime. In other words, in curved spacetime the continuous transformations of a purely time rate-of-change of a function of the space and time coordinates, from its frame of reference where it may appear by itself, to any other continuously connected frame of reference, leads to a mixture of space and time differential changes. In this case we cannot refer to an objective conservation (in time alone) of any quantity, in the curved spacetime. Therefore, the Noether construction becomes trivial and no conserved quantities can be found.

Energy-momentum conservation has only been *approximately* proved in GR for simple special cases involving spacetime regions of dimension small compared with the radii of curvature whereby the error is attributed to the gravitational field acting on the matter and itself having some energy and momentum.^{53,54} If the full, nonlinear GR is correct, then the door may be open to violating momentum conservation in FTL space warps and to the potential existence of "space drives".

VII. FTL Space Warps and Time Machines

A time machine is simply any closed timelike curve (CTC), not necessarily a geodesic, in spacetime. It is a well known and widely accepted fact that classical general relativistic and semi-classical quantum gravity theories are infested with time machines whereby there are numerous spacetime geometry solutions that exhibit time travel and/or have the properties of time machines.³¹ Traversable wormholes imply time machines, and this discovery spawned a number of follow-on research efforts on time machines.^{31,33,55-62} Given a traversable wormhole, it appears to be very easy to build a time machine, although it will require a Herculean effort to induce a time shift between the two wormhole mouths (via special relativistic or general relativistic time dilation techniques). Comprehensive theoretical studies show that the creation of a time machine might be the generic fate of traversable wormholes.³¹ Not surprisingly, warp drives also engender the appearance of CTCs.^{63,64}

Time machines are unavoidable in our physical (3+1)-dimensional spacetime. Einstein's GR admits solutions that violate causality, e.g., the Kerr solution, Gödel's universe, etc. K. S. Thorne⁶⁵ states that it may turn out that causality is violated at the macroscopic scale. Even if causality is obeyed macroscopically, then quantum gravity might offer finite probability amplitudes for microscopic spacetime histories possessing time machines. L.-X. Li and J. R. Gott⁶⁶ found a self-consistent vacuum for quantum fields in Misner space (a simple flat space with CTCs) for which the renormalized energy-momentum tensor is regular (in fact zero) everywhere. This implies that CTCs could exist at least at the level of semi-classical quantum gravity theory. Since the global properties of spacetime break causality in GR, this cannot be related to some intrinsic disease of superluminal motion.

A proper framework to better understand the interplay between causality and superluminal motion is discussed in Reference 2. In short, the causal light cones of special relativity have no special place over and above the light cones of any other system. This is not the traditional view which states that the special relativity causal light cones have a preferred role in physics which arises from the fact that a number of other systems (e.g., electromagnetic fields, spin-s fields, etc.) employ precisely those same light cones as their own. And it may be the case that the physical world is organized around such a commonality of causal light cones. On the other hand, it is entirely possible that there exist any number of other systems – not yet observed – that employ quite different sets of causal light cones. And the light cones of these other systems could very well lie outside the null light cones of special relativity, i.e., these systems could very well manifest superluminal signals and superluminal motion. None of this would

contradict our fundamental ideas about how physics is structured – an initial-value formulation, causal light cones governing signals, etc.

Time machines are now a noncontroversial mainstream topic of GR research.⁵⁵⁻⁶² It is our informed opinion that FTL space warp-time machines are not fatal, not destabilizing, or causally ill-behaved given the proper understanding of the logical interplay between causality, relativistic field Lorentzian metric local chronology, and superluminal motion. FTL space warp-time machines may be desirable for purposes other than space transportation.

VIII. FTL Space Warp Navigation, Guidance and Control

GR does not provide any mathematical recipes for FTL space warp navigation, guidance and control through space. FTL motion through space is different for warp drives and traversable wormholes. Star travelers using a warp drive move through space at apparent FTL speed via the spacetime distortion created by the warp bubble while they sit inside it. The situation is different for star travelers who use a traversable wormhole, because a wormhole is simply a hyperspace tunnel/shortcut through space and they must traverse it at sublight speed using Newtonian or relativistic rockets. Their motion in space between stars via a wormhole will appear to be FTL as seen by outside (remote) static observers.

The technical description of a trip through a spherically symmetric traversable wormhole is simply given by the proper time and the proper distance of travel through its throat as measured by space travelers who always move on timelike curves, while the (radial) travel velocity through the throat is $v = v(r) < c$. The proper time of travel as measured by space travelers going through the wormhole is given by $\Delta\tau = \int (\gamma v)^{-1} d\ell$, where $\gamma = [1 - (v/c)^2]^{-1/2}$ and the integration is taken from the wormhole entrance to its exit.⁶⁷ The proper distance of travel as measured by the space travelers is $\Delta\ell = v\Delta\tau$. Outside remote static observers watching the space travelers go through the wormhole will measure the travel time to be $\Delta t = \int (v e^{\phi(r)})^{-1} d\ell$, the travel distance will be $\Delta\ell = v\Delta t$, and the integration is taken over the same limits as before.⁶⁷ The Appendix in Reference 67 provides three case studies for traversable wormholes with different exotic matter and throat geometry specifications, and the corresponding travel times and distances are calculated. Whatever time-distance values one desires for their interstellar trip will depend largely on 1) the type of exotic matter that one chooses to assemble the wormhole; and 2) the particular type of wormhole throat geometry, which serves as further input to throat stability analysis, determining the tidal gravitational forces exerted on the starship crew along the way, etc.

In the case of warp drives, the starship moves on a timelike curve by design.⁶⁸ This trajectory establishes a very simple relationship between the element of coordinate time and the element of proper time, $d\tau = dt$. Because the coordinate time is also equal to the proper time of remote static observers in the flat-space region far away from the warp bubble, the starship suffers no relativistic time dilation as it moves. The proper acceleration along the starship's path (a geodesic) will always be zero while the coordinate acceleration can be an arbitrary function of time. For the case of a one-way trip from Earth to some stellar destination a distance D away, the total elapsed proper time as measured onboard the starship, which is equal to the total elapsed coordinate time T , is $\tau = T = 2\sqrt{D/a}$.⁶⁸ T can be made as small as one desires simply by increasing the value of the starship's acceleration a , which is how apparent FTL motion is manifested in warp drives.

The control laws, cockpit/bridge visual display, reference/guide stars, and star maps for navigation and guidance in interstellar space have not been developed for either type of FTL space warp. Navigation and guidance systems will have to be developed separately for warp drives and traversable wormholes because of their different implementations. D. G. Hoag and W. Wrigley⁶⁹ studied navigation and guidance for relativistic interstellar missions, and G. Vulpatti⁷⁰ studied relativistic navigation using the 3-dimensional rocket equation. These studies could provide a starting point to begin an equivalent study for FTL interstellar missions.

Of further interest is how the forward and aft starfields appear to starship crews who visually monitor their flight progress using electronic visual displays and/or windows during FTL flight or while traversing a wormhole. L. H. Ford and T. A. Roman¹² and C. Clark, W. A. Hiscock and S. L. Larson⁷¹ show that for a warp drive starship at FTL speed, the angular deflection and redshift of photons propagating through the distortion of the warp bubble is such that stars in the forward and reverse hemispheres will appear closer to the direction of motion than they would to an observer at rest. The stars in the forward direction will be strongly blueshifted and in the aft direction they will be strongly redshifted. The light from stars directly overhead, underneath or to the sides remains unaffected by the aberration. This aberration is qualitatively similar to that caused by SR for the case of relativistic rockets.⁷²⁻⁷⁴ This suggests that visual guide/reference stars and typical star maps will be useless for warp drive starship navigation.

Real-time electronic visual displays will be required to display accurate virtual starfields and maps, and they must have computer algorithms that perform real-time adjustments to account for the effects of FTL aberration in order to display visually meaningful views and maps.

The view through a traversable wormhole is even worse. The negative energy density threading a wormhole throat produces repulsive gravity, which deflects light rays going through and around the throat.⁷⁵ The entrance to the (spherically symmetric) wormhole would look like a sphere that contained the mirror image of a whole other universe or remote region within our universe, incredibly shrunken and distorted. This is a topological inversion of images manifested by a spherically symmetric wormhole geometry. The spherical wormhole entrance/exit (a.k.a. the throat) is called a hypersphere because it is the hyperspace surface of (3+1)-dimensional spacetime. If one were to travel through the wormhole and look back at it from the other side, then they would see a sphere (the entry way back home) that seemed to contain their whole original universe or their home region of space near Earth. This would look just like a glass Christmas tree ornament, which is just a spherical mirror that reflects, in principle, the entire universe around it. A flat-face traversable wormhole would not distort the image of the remote space region or other universe seen through it because the negative energy density at the throat is zero as seen and felt by light and matter passing through it.⁷⁶

W. A. Hiscock,⁴⁰ S. Finazzi, S. Liberati and C. Barcelo,⁴¹ C. Barcelo, S. Finazzi and S. Liberati,⁴² and C. Clark, W. A. Hiscock and S. L. Larson⁷¹ studied a modified Alcubierre warp drive in a toy (1+1)-dimensional spacetime and found that behind the (massless) starship a conical region forms from within which no signal can reach it, which is effectively an event horizon. Conversely, an event horizon-like structure in a conical region also forms in front of the starship into which it cannot send a signal. These structures are somewhat analogous to the Mach cones associated with supersonic fluid flow. The interior of the warp bubble thus becomes causally disconnected from its forward and reverse edges. The consequences of this are that the starship bridge cannot steer the warp bubble or turn it on/off, and information cannot be sent from the bridge to the region where the negative energy field needs to be controlled. Krasnikov's warp tube model attempts to cure this pathology. These investigators claim that the existence of horizon structures around the warp bubble suggests that the divergence of quantum vacuum energy when the starship moves FTL will likely also be present in physical (3+1)-dimensional spacetime, and prevent any warp drive starship from ever effectively moving FTL. However, no analysis of this problem has been done to reproduce these claimed effects in physical (3+1)-dimensional spacetime, and it may not be possible to do so. Therefore, the claimed effects are premature and potentially spurious.

IX. Conclusion and Next Steps

Implementing FTL space warps for interstellar flight appears to be as technically daunting as implementing relativistic rockets. But FTL space warps offer the potential of exceeding light speed while also drastically lowering the required flight time and distance as well as eliminate the adverse effects of relativistic time dilation. Exotic matter is required to construct and stabilize FTL space warps. Exotic matter typically has negative energy density and/or negative pressure associated with it. There is a large class of phenomenological and natural forms of classical and quantum exotic matter to consider for the purpose of constructing FTL space warps.² The following briefly outlines the technical issues that need to be addressed in order to continue evolving FTL space warps into a mature science and technology development program.

Next steps 1 – FTL Space Warp Computer Simulations: It would be valuable to simulate FTL space warps in the lab using analogue gravity-based experiments and computer model simulations to gain new insight into the problem of constructing and stabilizing FTL space warps and their associated physical effects. EarthTech Int'l, Inc. (parent of the Institute for Advanced Studies at Austin) and MetaRogue, Inc. are designing a transformational optics-analogue gravity experiment using metamaterials to simulate warp drives and traversable wormholes in the lab.^{77,78} This will also include exploring the use of computer simulation studies using recently developed codes for visualizing the nonlinear effects of spacetime curvature.⁷⁹

Next steps 2 – Quantum Inequalities: Further studies incorporating interacting fields in the QIs formulation should be implemented. However, extending the QIs to the physical (3+1)-dimensional spacetime doesn't appear practicable until new mathematical tools or a universally accepted quantum theory of gravity become available. The variety of Casimir effects, observational cosmology, and black holes appear to contradict the efficacy of the QIs. There are other phenomenological and natural afflictions that were cited earlier. Lab experiments should be conducted to test the QIs (see Next steps 9).

Next steps 3 – Quantum Back-Reaction: Quantum back-reaction is a nascent field of study, and so it will not be possible to say anything definitive in the foreseeable future about its full effects on the technical feasibility of FTL space warps in physical (3+1)-dimensional spacetime. However, calculations should be done to determine the

appropriate local radius of curvature for the varieties of traversable wormholes and warp drives to obtain accurate values for the negative vacuum polarization energy produced by their corresponding back-reaction, which would also provide a useful gauge for the magnitude of non-local particle production.

Next steps 4 – Conservation of Momentum in FTL Space Warps: No further work is recommended on this topic unless or until a universally accepted alternative theory of gravity or quantum theory of gravity is developed which can hopefully resolve the issue of quantifying gravitational energy-momentum. However, theoretical studies should be undertaken to attempt to define a class of GR-derived or even semi-classical quantum gravity-derived space drives. This might include an in-depth study of the meaning of inertia and mass in curved spacetimes and their associated quantum effects. Exploring emergent spacetime/gravity theories may offer additional insights.^{80,81}

Next steps 5 – FTL-Induced Time Machines: No further work is recommended on this topic until a traversable wormhole or a warp drive is implemented and tested in the lab. At that time, an empirical study would be better suited to ascertain the nature of the associated CTCs and how to mitigate them. Time travel is not germane to the goals of interstellar flight. However, some FTL starship travelers may find it desirable to avail themselves of FTL space warp-induced time travel for interesting exploration alternatives.

Next steps 6 – FTL Guidance, Navigation and Control: A theoretical program should be implemented to define the principles of navigation, guidance and control for traversable wormholes and warp drives in physical (3+1)-dimensional spacetime, and to determine what is possible to do under different scenarios and conditions. Empirical studies could test the results of this study should either type of FTL space warp be invented in the lab.

Next steps 7 – Alternative Theories of Gravity: It is recommended that the present genre of alternative gravity theories and quantum gravity theories be explored to determine whether they allow warp drive solutions. This would be desirable for the purpose of gathering new insights on warp drive physics that could come from such inquiries. Also, further inquiry along this line should continue for traversable wormholes because new insights on their physics are beginning to emerge. Also, the genre of emergent spacetime/gravity theories should be explored for the same reasons.^{80,81}

Next steps 8 – Producing Negative Energy: The Casimir effect is produced by a broad range of different materials and cavity geometries⁸² and other quantum field theoretic effects which are still under study.⁸³ The most important factor leading to negative energy densities in the Casimir effect is mode exclusion of the vacuum zero-point fluctuations. It is possible that new methods or new boundary conditions or interacting quantum fields will be found that can potentially produce large amounts of negative energy. The Ford-Svaiter cylindrical parabolic mirror should be parametrically evaluated to ascertain how much negative energy it produces, whether it is scalable, and whether it is feasible to engineer and test in the lab. The engineering parameter space for the quantum optical devices that produce squeezed vacuum states should be explored to discover whether new implementations can produce and isolate negative energy in large amounts. It is recommended that a concentrated multi-tiered theoretical and experimental research program be initiated to address all of these issues.

Next steps 9 – Detecting Negative Energy: A study should be initiated to test the Davies-Ottewill analysis using quantum optical tomography in order to elucidate the response of physical particle detectors to laboratory sources of negative energy densities/fluxes. EarthTech Int'l, Inc. and P. Marecki are in the planning stages of developing Marecki's proposed BHD-Casimir cavity experiment for this purpose.

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