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The gravitational mechanism to generate mass II

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With the eminent confirmation or disproof of the existence of Higgs boson by experiments on the LHC it is time to analyze in a non-dogmatic way the suggestions to understand the origin of the mass. Here we analyze the recent proposal according to which gravity is what is really responsible for the generation of mass of all bodies. The great novelty of such mechanism is that the gravitational field acts merely as a catalyst, once the final expression of the mass does not depend either on the intensity or on the particular characteristics of the gravitational field.

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I. INTRODUCTION: RECIPE FOR GENERATING MASS

In order to become a reliable candidate as a mechanism to generate mass, there are three indispensable conditions that such mechanism has to fulfil, to wit:

- There must exist a universal field that interacts with all kinds of particles;
- There must exist a free parameter such that different bodies can acquire distinct values for their corresponding mass (the spectrum of mass);
- This field must be such that its interaction with matter breaks explicitly some symmetry that only massless particles exhibit, e.g. the gauge freedom for vector fields or the chirality for fermions.

There are only two fashionable candidates that fulfill the first condition:

- A scalar field φ;
- The gravitational field.

The Higgs boson φ was postulated to couple universally with all kinds of matter. However, still to this day there is no evidence of its universality, put aside its own existence[1]. The other one, gravity, is known to couple with all forms of matter and energy; its universality is recognized as a scientific truth. We note that after accepting either one of these two fields as a good candidate that fulfills the first requirement, it is not a hard job to elaborate scenarios such that the other two conditions are satisfied too.

In this work we will limit our analysis to the gravitational process [2], [3] once in the realm of high-energy physics, the Higgs model produced a well-known alternative scenario for generating mass for all massive particles except the Higgs boson itself [4]. Let us just point out a remarkable property of the Higgs mechanism that within its scenario was not sufficiently emphasized. It concerns the property that the self-interacting scalar field in order to generate the mass of the particles must be in its fundamental state. Its energy distribution is described as

$$T_{\mu\nu} = V(\varphi_0)\, g_{\mu\nu}$$

defining a cosmological constant $V(\varphi_0)$. However, in this mechanism, this fact is no further analyzed, since at the realm of microphysics gravity is ignored. So much for this structure. Let us turn now to the new mechanism.

II. NUMERICAL RESULTS

Before entering in the details of the gravitational mechanism let us point out some of its observational consequences. We start by recalling that the inverse Compton length of any particle is given in terms of its mass M, the Planck constant $\hbar$ and light velocity c yielding

$$\mu = \frac{c}{\hbar}\, M.$$

For latter use we re-write it in an equivalent way in terms of gravitational variables using the Newton constant G_N or equivalently the Einstein constant $\kappa = 8\pi\, G_N/c^4$. The Schwarzschild solution of the gravitational field of a static compact object has an horizon – that is a one-way membrane – characterized by its Schwarzschild radius

$$r_s = \frac{1}{4\pi}\,\kappa\, M\, c^2$$

Using the definition of the Planck length

$$L_{Pl}^2 \equiv \frac{1}{8\pi}\,\kappa\,\hbar\, c$$

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[1] See [1] for alternative exotic scenarios suggested to take the place of Higgs mechanism in case discovery of Higgs boson fails.

it follows that inverse Compton length may be written under the equivalent form as the ratio between the corresponding Schwarzschild radius and the square of Planck length:

$$\mu = \frac{1}{2}\,\frac{r_s}{L_{Pl}^2}. \tag{1}$$

The formula of the mass we obtained in [2] (and which we will review in the next section) from the non-minimal coupling of a spinor field Ψ with gravity is expressed in terms of the cosmological constant Λ, the Planck length and parameter σ of the non-minimal coupling yielding the expression

$$\mu = \frac{1}{8\pi}\,\frac{\sigma\,\Lambda}{L_{Pl}^2} \tag{2}$$

This expression relates two parameters: the mass M and the associated non-minimal coupling constant with gravity σ that has the dimensionality of a volume. The knowledge of one of these two parameters (M and σ) allows the knowledge of its companion. By comparison of the above two expressions of μ, that is, Compton definition eq. (1) and our formula for the mass eq. (2) yields the expression of σ :

$$\sigma = 4\,\pi\,\frac{r_s}{\Lambda} \tag{3}$$

Thus different fermion particles that have different masses have different values of σ. We note furthermore that the ratio M/σ which has the meaning of a density of mass is a universal constant given only in terms of κ and Λ. How to interpret such universality? There is a direct and simple way that is the following. We re-write this formula as a density of energy, that is

$$\frac{M\,c^2}{\sigma} = \frac{\Lambda}{\kappa} \tag{4}$$

The right-hand side is nothing but the density of energy of the vacuum. Thus we can say that σ is the volume in which an homogeneous distribution of the particle energy spreads having the same value of the vacuum energy density provided by the cosmological constant, that is, Λ/κ.

Once our formula of mass for fermions contains gravitational quantities which are well-known to be extremely small, let us compare it with actual numbers that we can get, for instance, from the simplest example of the electron. The main question is: should the coupling constant σ become an enormously big value in order to compensate the weakness of the gravitational field? A direct calculation for the known elementary particles show that this is not the case. This is a direct consequence, as we shall see in the next section, of the fact that, in the process of give mass, gravity enters only as a catalyst. Indeed, for the simple stable lepton, the electron we find that

$$r_s \approx 1.35 \times 10^{-55} cm,$$

which implies[2] that

$$\sigma_e \approx 125\, cm^3.$$

The substance that we call electron is tremendously concentrated within its Compton wavelength λ_c. Indeed if we compare the density of energy $M_e\,c^2/r^3$ for $r = \lambda_c$ and σ, it follows that all of the electron is concentrated in its Compton interior:

$$\frac{\varrho_c}{\varrho_\sigma} \approx 10^{31}.$$

Before ending this section let us make a remark in order to test the coherence of our formula (4) in the cosmological scenario. Indeed, suppose the extremal case identifying σ with the total volume of the Universe. Assuming that the universe is roughly made by protons, we can estimate the total number of protons N_p in the universe. A direct calculation using equation (4) for $\sigma \approx (10^{28} cm)^3$, yields

$$N_p \approx 10^{80} \text{ protons.}$$

We note that this number is precisely Eddington number.

III. MINIMAL MASS VALUE

The present method of evaluating the mass takes into account only classical gravitational aspects. Thus, in principle it stops to be applied at the quantum level. Indeed, quantum effects become non negligible at least at the Compton wavelength of a given particle. This means that there is a threshold of applicability of our mechanism. In other words the value of the length associated to the gravitational mechanism must be higher than the corresponding Compton wavelength of the particle. This led naturally to the minimum value of the mass of any fermion - call it M_q - that can be generated by the gravitational procedure. In other words we must have $\sigma \geq \lambda_c^3$, that is

$$M_q c^2 \geq \frac{\Lambda}{\kappa}\,\frac{\hbar^3}{M_q^3 c^3},$$

from which we obtain that the minimum possible value for the mass is

$$M_q \geq 2.36 \times 10^{-3}\, eV.$$

Thus there is no possibility of having a fermion which has a mass lower than M_q.

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[2] All the values used here were taken from the *Particle Data Group* [6].

IV. FROM MACH PRINCIPLE TO THE NEW GRAVITY MECHANISM

Although a widespread formulation — identified as Mach's principle – that the mass of a body may depend on the overall properties of the rest-of-the-universe and consequently to gravity, the association of this dependence to the smallness of gravitational phenomena was at the origin of the general attitude of disregarding any possibility to attribute to gravity an important role in the generation of mass for all bodies (see however [5]).

This apparent difficulty is eliminated by two steps:

- A direct coupling of matter to the curvature of space-time;
- The existence of a vacuum distribution or cosmological constant Λ.

This idea was developed recently [2] thus providing a reliable mechanism by means of which gravity is presented as truly responsible for the generation of the mass. As a result of such procedure, the final expression of mass depends neither on the intensity nor on the specific properties of the gravitational field. This circumvents all previous criticism against the major role of gravity in the origin of mass.

The model uses a slight modification of Mach's principle. Let us remind that, following Einstein [7], we can understand by this principle the statement according to which the entire inertia of a massive body is the effect of the presence of all other masses, deriving from a kind of interaction with the latter or, in other words, the inertial properties of a body $\mathbb{A}$ are determined by the energy throughout all space. The simplest way to implement this idea is to consider the state that takes into account the whole contribution of the rest-of-the-universe onto $\mathbb{A}$ as the most homogeneous one. Thus it is natural to relate it to what Einstein attributed to the cosmological constant or, in modern language, the vacuum of all remaining bodies. This means to describe the energy-momentum distribution of all complementary bodies of $\mathbb{A}$ in the Universe under the form

$$T_{\mu\nu}(U) = \frac{\Lambda}{\kappa} g_{\mu\nu} \tag{5}$$

Note that this distribution of the energy content of the environment of the body $\mathbb{A}$ is similar to the Higgs case, although there is an important distinction concerning the role of this homogeneous distribution of energy on the generation of mass: as we pointed out above, Higgs'proposal does not go further to explore the consequences of this distribution of energy, since it is not followed by the analysis relating such energy to gravitational processes. We consider the very fundamental framework dealing with the basic constituents of matter, the true building blocks, and treat matter generically as representations of the Lorentz group. In the present paper we limit our description to the case in which body $\mathbb{A}$ is identified with fermions.

A. The case of fermions

The massless theory for a spinor field is given by Dirac equation:

$$i\gamma^\mu \partial_\mu \Psi = 0 \tag{6}$$

This equation is invariant under γ^5 transformation. In order to have mass for the fermion this symmetry must be broken. Who is the responsible for this? Electrodynamics appears in gauge theory as a mechanism that preserves a symmetry when one pass from a global transformation to a local one (space-time dependent map). Nothing similar with gravity. Following [2] we assume the idea that gravity is the true responsible to break the symmetry.

In the framework of General Relativity the non-minimal gravitational interaction of the fermion is driven by the Lagrangian

$$L = L_D + L_{int} + L_\Lambda + L_{ct}$$

that is

$$\begin{aligned} L &= \frac{i}{2}\bar{\Psi}\gamma^\mu\nabla_\mu\Psi - \frac{i}{2}\nabla_\mu\bar{\Psi}\gamma^\mu\Psi \\ &+ \frac{1}{\kappa}(1+\frac{\sigma}{4}\Phi)^{-2}R - \frac{1}{\kappa}\Lambda \\ &- \frac{3}{8\kappa}\sigma^2(1+\frac{\sigma}{4}\Phi)^{-4}\partial_\mu\Phi\,\partial^\mu\Phi, \end{aligned} \tag{7}$$

where the non-minimal coupling of the spinor field with gravity is contained in the term $V(\Phi) = 1 + \sigma\,\Phi/4$ that depends on the scalar

$$\Phi \equiv \bar{\Psi}\,\Psi,$$

which preserves the gauge invariance of the theory under the map $\Psi \to \exp(i\,\theta)\,\Psi$. Note that the dependence on Φ on the dynamics of Ψ breaks the chiral invariance of the mass-less fermion, a condition that is necessary for a mass to appear. The constant σ which has dimensionality $(length)^3$ given by (3) is the responsible for the non-minimal coupling and the presence of the self-interacting term.

This dynamics represents a massless spinor field coupled non-minimally with gravity. The cosmological constant represents the influence of the rest-of-the-universe on Ψ.

Independent variation of Ψ and $g_{\mu\nu}$ yields

$$i\gamma^\mu\nabla_\mu\,\Psi + \Sigma\,\Psi = 0, \tag{8}$$

$$\alpha_0\,(R_{\mu\nu} - \frac{1}{2}\,R\,g_{\mu\nu}) = -T_{\mu\nu}, \tag{9}$$

where Σ depends on the curvature scalar R and on Φ.

The energy-momentum tensor is defined by

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\, L)}{\delta g^{\mu\nu}}.$$

Taking the trace of equation (9) and inserting it on the expression Σ one obtains after some algebraic manipulation[3] that the equation for the spinor becomes

$$i\gamma^{\mu}\nabla_{\mu}\,\Psi - M\Psi = 0, \tag{10}$$

where

$$M = \frac{\sigma\,\Lambda}{\kappa\, c^2}. \tag{11}$$

Thus as a result of the above process the spinor field acquires a mass M that depends crucially on the existence of Λ. If Λ vanishes then the mass of the field vanishes. The non-minimal coupling of gravity with the spinor field corresponds to a specific self-interaction. The mass of the field appears only if we take into account the existence of all remaining bodies in the universe — represented by the cosmological constant. The values of different masses for different fields are contemplated in the parameter σ.

This procedure allows us to state that the mechanism proposed here is to be understood as a realization of Mach principle according to which the inertia of a body depends on the background of the rest-of-the-universe. This strategy can be applied in a more general context in support of the idea that (local) properties of microphysics may depend on the (global) properties of the universe. In the case $\sigma = 0$ the Lagrangian reduces to a massless fermion satisfying Dirac's dynamics plus the gravitational field described by General Relativity.

B. A new interpretation of the non-minimal coupling

There is another interpretation of the Lagrangian (7) that is worth to point out here because it shows that the non-minimal coupling described by (7) can be interpreted as the conformal coupling of a scalar field with gravity. Let us define the non dimensional scalar field X by setting

$$X = \frac{1}{\sqrt{6}}(1 + \frac{\sigma}{4}\,\Phi)^{-1}$$

Then, in terms of this new quantity the dynamics (7) can be re-written as

$$L = L_D - \frac{\Lambda}{\kappa} - \frac{1}{\kappa}(\partial_{\mu}X\,\partial^{\mu}X - \frac{1}{6}\,R\,X^2), \tag{12}$$

which is nothing but Dirac dynamics plus the equation of a scalar field X coupled in a conformal way to the curvature of space-time. We recognize here the standard procedure of conformal coupling a scalar field with gravity. When the field X is identified with the chiral dependent term constructed with the spinor field through the above definition then the net effect of gravity through the existence of a cosmological constant appears and provides mass for Ψ.

V. MODIFIED MACH PRINCIPLE

The various steps of our mechanism driven by Lagrangian (7) are synthesized as follows:

- The dynamics of a massless spinor field Ψ interacting to gravity in a conformal way is contained in the Lagrangian

$$L = L_D - \frac{\Lambda}{\kappa} - \frac{1}{\kappa}\,(\partial_{\mu}X\,\partial^{\mu}X - \frac{1}{6}\,R\,X^2);$$

- Gravity is described by General Relativity;

- The action of the rest-of-the-universe on the spinor field, through the gravitational intermediary, is contained in the form of an additional constant term on the Lagrangian noted as Λ;

- As a result of this process, the field Ψ acquires a mass M given by expression (11) and is zero only if the cosmological constant vanishes;

- This process is completely independent from the intensity and the specific configuration of the gravitational field.

The generalization of this procedure for all other kinds of matter which are representations of the Lorentz group (scalar or tensor fields) can be made straightforwardly through the same lines as in the present case.

The mechanism presented in this paper allows us to interpret the mass of any body as being nothing but a local property originated by the influence of the whole universe intermediated by the gravitational interaction. In other words, to explain the origin of mass for all bodies, there is no need to introduce extra fields as, for instance, the Higgs boson.

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