Resonant cavities and space-time

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Abstract

I analyze the behavior of electromagnetic fields inside a cavity by solving Einstein field equations. It is shown that the modified geometry of space-time inside the cavity due to a propagating mode can affect the propagation of a laser beam. The effect is the appearance of components of laser light with a shifted frequency originating from the coupling between the laser field and the mode cavity due to gravity.

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I. INTRODUCTION

A single plane wave always induces a deformation of the geometry of the space-time [1]. This effect is so small and plane waves such idealized objects that hopes to observe it are certainly very tiny. Anyway, electromagnetic fields are easily available and the technology is old and it is not impossible to realize devices where the intensity of such fields could make this gravitational effect observable. This entails a rather sensible interferometer but it is not impossible to realize. The devices that better fit the aim are resonant cavities where, due to large merit factors, gross intensity of the electromagnetic energy can be achieved. The observation of such an effect would mean a real breakthrough in experimental general relativity as, so far, only large scale measurements were considered possible and so available. A table-top experiment would completely change our way to manage gravitational fields and could pave the way to a possible engineering of space-time due to our ability to manage and produce electromagnetic fields.

In this paper I show a simple textbook computation in general relativity showing how a resonant cavity with a single mode excited inside could provide a satsfactory set-up for such a measurement and could explain the recent measurements done at Eagleworks using a frustum as a resonator. This should be considered just the starting point for a more extended treatment to the experimental set-up, much on the same lines of Ref.[2]. In the latter paper a modified Einstein theory was considered but, with proper adjustments, the computations could easily fit the bill in our case.

II. PLANE WAVE GEOMETRY

A. Geometry

The simplest case discussed in literature for the Einstein-Maxwell equations is that of a plane wave [1]. I take the metric in the form

$$ds^2 = L^2(v)(dx^2 + dy^2) - dvdu \tag{1}$$

given the Rosen coordinates $v = ct - z$ and $u = ct + z$. It is easy to show that an electromagnetic plane wave modifies the geometry of space-time. I have that the Einstein tensor reduces to the Ricci tensor as the trace of the energy-momentum tensor is zero in this case.

I will have the only non-null component

$$R_{33} = -2\frac{L''(v)}{L(v)}. \quad (2)$$

The electromagnetic field tensor will have the non-null components

$$F_{31} = -F_{13} = A'(v). \quad (3)$$

So, the only nonzero component of the energy-momentum tensor is

$$T_{33} = -\frac{1}{\mu_0}\frac{(A'(v))^2}{L^2(v)} \quad (4)$$

and so I have to solve the equation

$$2\frac{L''(v)}{L(v)} = -\frac{8\pi G}{c^4\mu_0}\frac{(A'(v))^2}{L^2(v)} \quad (5)$$

That has the solution $L(v) = \pm\alpha A'(v)$ provided

$$\alpha^2 = \frac{4\pi G}{c^2\omega^2\mu_0} \quad (6)$$

and I am left with the equation for a plane wave

$$[A'(v)]'' + \frac{\omega^2}{c^2}A'(v) = 0 \quad (7)$$

for the electromagnetic field and taking $A'(0) = E_0/c$ the magnetic field amplitude. Note that $\alpha \approx 9 \cdot 10^{-21}\ A \cdot m \cdot N^{-1} = 9 \cdot 10^{-21}\ T^{-1}$ for $\omega = 1\ GHz$. This is a small number as expected and this effect is negligible small for all practical purposes. Its inverse identify a critical magnetic field for which this effect could be meaningful but has an unphysical large value.

In a resonant cavity, an estimation of the amplitude of the electric field E_0 can be computed using the formula [2]

$$\frac{\epsilon_0}{4}E_0^2L^3 = \frac{Q \cdot P}{\omega} \quad (8)$$

being Q the merit factor, P the input power and V the volume of the cavity assumed to be a box of side length L. In this case I have to apply the boundary condition

$$A'(0) = A'(L). \quad (9)$$

This yields the modes to be $k_n = n\pi/L$ and the corresponding frequencies $\omega_n = ck_n$ arising from the Rosen coordinate $v = ct - z$.

B. Light propagation

I assume that a beam of light is moving through the box containing the mode described above as the cavity is fed through some source. There is no electromagnetic interaction between these two electromagnetic fields because light has not self-interaction besides a small effect, dubbed Delbrück scattering, that can be analyzed in quantum electrodynamics and is fourth order. This competes with the gravitational correction. The propagation of the beam inside the cavity is described by the wave equation

$$L^2(v)\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right)-4\frac{\partial^2\psi}{\partial u\partial v}=0 \tag{10}$$

and one sees that the altered geometry by the mode of the cavity can couple it with the laser beam. This equation can be solved by separation of variables setting

$$\psi(x,y,u,v)=\mathcal{E}(x,y)\phi(u,v) \tag{11}$$

being $\mathcal{E}(x,y)$ an envelope of the beam. This yields the equation for $\phi(u,v)$

$$-4\frac{\partial^2\phi}{\partial u\partial v}=k^2L^2(v)\phi \tag{12}$$

that is

$$\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}-\frac{\partial^2\phi}{\partial z^2}=k^2L^2(ct-z)\phi. \tag{13}$$

One can consider $L(ct-z)$ a small quantity and do some perturbation theory yielding

$$\phi(z,t)\approx\phi_0(z,t)+\frac{ck^2}{2}\int dz'dt'\theta(c(t-t')-(z-z'))L^2(ct'-z')\phi_0(z',t') \tag{14}$$

being $\theta(z)$ the Heaviside step function and $\phi_0(z,t)$ the laser beam entering the cavity. Finally, one has

$$\psi(x,y,z,t)\approx\psi_0(x,y,z,t)+\frac{ck^2}{2}\int dz'dt'\theta(c(t-t')-(z-z'))L^2(ct'-z')\psi_0(x,y,z',t'). \tag{15}$$

One sees that there is an additional component to the laser field exiting the cavity that interacts with the mode inside. This can have terms with the frequency shifted and is a purely gravitational effect. In order to see this just note that

$$L^2(ct-z)=\alpha^2\frac{E_0^2}{2c^2}\left(2+e^{i\omega(t-z/c)}+e^{-i\omega(t-z/c)}\right) \tag{16}$$

and, for the laser field,

$$\psi_0(x,y,z,t) = A(x,y,z)e^{i\omega_L t} + A^*(x,y,z)e^{-i\omega_L t}. \tag{17}$$

Putting this into eq.(15) one sees that the additional components contribute as

$$\psi(x,y,z,t) \approx \psi_0(x,y,z,t) + \frac{k^2}{4}\alpha\frac{E_0^2}{c}\left(A_1(x,y,z)e^{i\omega_L t} + A_2(x,y,z)e^{i(\omega-\omega_L)t} + A_3(x,y,z)e^{i(\omega+\omega_L)t} + c.c.\right). \tag{18}$$

One should observe satellite lines due to the modified geometry of space-time originating from the field inside the cavity.

III. CONCLUSIONS

I have shown how a plane wave could produce a gravitational effect inside a cavity that could be observed using a propagating laser beam inside it. The effect could be unveiled using an interferometer or observing the components of the laser field outside the cavity. Components with a shifted frequency, due to the modes inside the cavity, should be seen. This could explain recent results at Eagleworks with a resonator having the form of a truncated cone. A local warp of the geometry due to the electromagnetic field pumped inside the cavity could be a satisfactory explanation. From a physical standpoint this could be a really breakthrough paving the way to table-top experiments in general relativity and marking the starting point of space-time engineering.

The next step will be a full analysis of the truncated cone resonator. One should expect more contributions to the shifted frequency components of the laser field coming out from the cavity. It should be easier in this case to get evidence of this effect.

[1] C. W. Misner, K. S. Thorne and J. A. Wheeler, "Gravitation," San Francisco 1973, 1279p.

[2] F. O. Minotti, Grav. Cosmol. **19**, 201 (2013) [arXiv:1302.5690 [gr-qc]].