

It is this background independence that makes gravity and GRT fundamentally different from all other fields. And it is the reason why “frame dragging” is fully equivalent to the action of a gravitational force. If you want to preserve the configuration of a system before some nearby objects are accelerated, when the nearby objects begin to accelerate you have to exert a force that counteracts the effect of the frame dragging produced by the acceleration of the nearby objects. When you do that, what do you feel? An inertial reaction force – the force produced by the action of the dragged spacetime, which is produced by the gravitational action of the accelerated nearby objects. By interposing frame dragging we’ve made it appear that no gravitational force is acting. But of course gravity is acting, notwithstanding that we’ve introduced the intermediary of frame dragging to make it appear otherwise.

When only nearby objects are accelerated to produce frame dragging, as Einstein noted for the equivalent force he expected, the predicted effects are quite small. When it is the universe that is accelerated, it is the full normal inertial reaction force that is felt if you constrain some object to not accelerate with the universe. Why the difference? Because when the entire universe is “rigidly” accelerated, the interior spacetime is rigidly dragged with it, whereas nearby objects, even with very large masses, produce only small, partial dragging.

You may be thinking, yeah, right, rigidly accelerating the whole universe. That would be a neat trick. Getting the timing right would be an insuperable task. The fact of the matter, nonetheless, is that you can do this. We all do. All the time. All we have to do is accelerate a local object. Your fist or foot, for example. The principle of relativity requires that such local accelerations be equivalent to considering the local object as at rest with the whole universe being accelerated in the opposite direction. And the calculation using the PPN formalism for frame dragging (with GRT values for the coefficients in the equation assumed) bears this out. At the end of his paper on gravimagnetism Nordtvedt showed that a sphere of radius  $R$  and mass  $M$  subjected to an acceleration  $\mathbf{a}$  drags the inertial space within it as:

$$\delta\mathbf{a}(\mathbf{r}, t) = -\left(2 + 2\gamma + \frac{\alpha_1}{2}\right) \frac{U(\mathbf{r}, t)}{c^2} \mathbf{a} \quad (2.8)$$

where the PPN coefficients have the values  $\gamma = 2$  and  $\alpha_1 = 0$  for the case of GRT and  $U(\mathbf{r}, t)$  is the Newtonian scalar potential, that is,  $U = GM/R$ . So we have four times  $\phi$  (changing back to the notation of Sciama’s work on Mach’s principle) equal to  $c^2$  to make  $\delta\mathbf{a} = \mathbf{a}$  in Eq. 2.8; that is, if the universe is accelerated in any direction, spacetime is rigidly dragged with it, making the acceleration locally undetectable.

You may be concerned by the difference of a factor of 4 between the Nordtvedt result and Sciama’s calculation. Factors of 2 and 4 are often encountered when doing calculations in GRT and comparing them with calculations done with approximations in, in effect, flat spacetime. In this case, resolution of the discrepancy was recently provided by Sultana and Kazanas, who did a detailed calculation of the contributions to the scalar potential using the features of modern “precision” cosmology (including things like dark matter and dark energy, and using the particle horizon rather than the Hubble sphere), but merely postulating the “Sciama force,” which, of course, did not include the factor of 4 recovered in Nordtvedt’s calculation. They, in their relativistically correct

Indeed, “persistent illusion” was exactly the way Einstein characterized our notions of past, present, and future and the passage of time to the relatives of his lifelong friend Michel Besso after Besso’s death, shortly before his own. The past and the future are really out there. Really. Not probably. You may think that this must all be a lot of nonsense dreamed up by people who don’t have enough real work to fill their time. But let me point out that if absurdly benign wormholes are ever to be built and actually work, then this worldview *must* be correct. The past and the future must really “already” be out there. How can you travel to a past or future that doesn’t “already” exist?

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low: We must make the world view correct, so as to enable benign absurdity? We no get it.

## THE “RELATIONAL” AND “PHYSICAL” VERSIONS OF MACH’S PRINCIPLE

Should you find the forgoing confusing and contentious, you’ll doubtless be disappointed to learn that we haven’t yet covered the full range of arguments involving Mach’s principle. As arguments about Mach’s principle developed over the decades of the 1950s, 1960s, and 1970s, two distinct ways of “interpreting” the principle emerged. One came to be called the “relationalist” view, and the other we shall call the “physical” view.

Serious arguments about Mach’s principle ceased to be fashionable in the mid-1970s. A few hardy souls wrote about the principle in the late 1970s and 1980s, but no one paid them much mind. Mach’s principle became fashionable again in the early 1990s, and Julian Barbour and Herbert Pfister organized a conference of experts in the field held in Tübingen in the summer of 1993. The proceedings of the conference were published as volume six of the *Einstein Studies* series with the title: *Mach’s Principle: From Newton’s Bucket to Quantum Gravity* (Birkhauser, Boston, 1994). This is an outstanding book, not least because the questions, comments, and dialog were published, as well as the technical papers presented.

Both the relationalist and physical positions on Mach’s principle were on display at the conference. Many of the attendees seem to have been convinced relationalists. The essence of the relationalist position is that all discussion of the motion of massive objects should be related to other massive objects; that relating the motion of objects to spacetime itself is not legitimate. This probably doesn’t sound very much like our discussion of Mach’s principle here. That’s because it isn’t. The relationalist approach says nothing at all about the origin of inertial reaction forces. The physical view of Mach’s principle, however, does. After the conference, one of the leading critics of Mach’s principle, Wolfgang Rindler, wrote a paper alleging that Mach’s principle was false, for it led to the prediction of the motion of satellites in orbit around planets that is not observed – that is, the motion was in the opposite direction from that predicted by GRT. It was 3 years before Herman Bondi and Joseph Samuel’s response to Rindler was published. They pointed out that while Rindler’s argument was correct, it was based on the relationalist interpretation of Mach’s principle. They argued that the physical interpretation that they took to be exemplified by GRT and Sciama’s model for inertia gave correct predictions. Therefore, Mach’s principle could not be dismissed as incorrect on the basis of satellite motion, as Rindler had hoped to do. It seems that Einstein was right in 1922, and Pais in 1982, when they remarked that Mach’s principle was a missing piece of the puzzle of the origin of inertia. We should now know better. After all, the WMAP results show that *as a matter of fact* space is flat, and it is certainly not empty, so if the principle of relativity, introduced by

Since the advent of GRT, those scientists interested in gravity have pretty much ignored Newtonian gravity. After all it is, at best, just an approximation to Einstein's correct theory of gravity. Engineers, however, have worked with Newtonian gravity all along. If you are doing orbit calculations for, say, a spacecraft on an interplanetary mission, the corrections to Newtonian mechanics from GRT are so utterly minuscule as to be irrelevant for practical purposes. Why engage in lengthy, tedious, and complicated calculations using GRT and increase the risk of miscalculation when the Newtonian gravity approximation is more than sufficient? True, the same cannot be said in the case of GPS calculations because the timing involved is more than precise enough to make GRT corrections essential. For your vacation trip, or shopping downtown, being off by as much as a 100 m or more is likely inconsequential. But if you are trying to blast the bunker of some tin-horned dictator, getting the position right to less than a few meters does make a difference (unless you are using a tactical nuke, in which case being off by up to half a kilometer probably won't matter).

## RELATIVISTIC NEWTONIAN GRAVITY

The relativistic version of Newtonian gravity gets mentioned in texts, but the field equations for relativistic Newtonian gravity do not get written out in standard vector notation as a general rule. Why bother writing down something that's a crude approximation? Nonetheless, George Luchak did so in the early 1950s, when constructing a formalism for the Schuster-Blackett conjecture. The Schuster-Blackett conjecture asserts that rotating, electrically neutral, massive objects generate magnetic fields. Were this true, it would couple gravity and electromagnetism in a novel way.

Luchak was chiefly interested in the anomalous coupling terms that get added to Maxwell's equations if the conjecture is true, so relativistic Newtonian gravity was a good enough approximation for him. Accordingly, contrary to established custom, instead of using the four dimensions of spacetime for tensor gravity and adding a fifth dimension to accommodate electromagnetism, he wrote down the equations of electromagnetism in the four dimensions of spacetime, and in the fifth dimension he wrote out a vector formalism for the scalar Newtonian approximation of relativistic gravity. Along with the curl of the gravity field  $\mathbf{F}$  being zero, he got two other equations, one of which being of sufficient interest to be worth writing out explicitly:

$$\nabla \bullet \mathbf{F} + \frac{1}{c} \frac{\partial q}{\partial t} = -4\pi\rho. \quad (3.1)$$

$\rho$  is the matter density source of the field  $\mathbf{F}$ , and  $q$  is the rate at which gravitational forces do work on a unit volume. (The other equation relates the gradient of  $q$  to the time rate of change of  $\mathbf{F}$ .)<sup>1</sup> The term in  $q$  in this equation appears because changes in gravity now propagate at the speed of light. It comes from the relativistic generalization of force,

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<sup>1</sup> For formalphiles, the equations are:  $\nabla \times \mathbf{F} = 0$  and  $\nabla \bullet q + \frac{1}{c} \frac{\partial \mathbf{F}}{\partial t} = 0$ .

namely, that force is the rate of change in proper time of the four-momentum, as discussed in Chap. 1. The time-like part of the four-force is the rate of change of  $mc$ , and since  $c$  is a constant in SRT (and strictly speaking, a locally measured invariant), that is just the rate of change of  $m$ , which is the rate of change of  $E/c^2$ . If  $m$  were a constant, too, this term would be zero. But in general  $m$  is not a constant.

A serious mistake is possible at this point. It is often assumed that the rest masses of objects are constants. So, the relativistic mass  $m$  can be taken as the rest mass, usually written as  $m_0$ , multiplied by the appropriate “Lorentz factor,” an expression that appears ubiquitously in Special Relativity Theory (SRT) equal to one divided by the square root of one minus the square of the velocity divided by the square of  $c$ .<sup>2</sup> The symbol capital Greek gamma is commonly used to designate the Lorentz factor. (Sometimes the small Greek gamma is used, too.) When  $v$  approaches  $c$ , the Lorentz factor, and concomitantly the relativistic mass, approaches infinity. But the proper mass is unaffected. If the proper mass  $m_0$  really is a constant, then the rate of change of  $mc$  is just the rate of change of the Lorentz factor. As Wolfgang Rindler points out in section 35 of his outstanding book on SRT, *Introduction to Special Relativity*, this is a mistake. It may be that in a particular situation rest mass can be taken as a constant. In general, however, this is simply not true. In a situation as simple as the elastic collision of two objects, during the impact as energy is stored in elastic stresses, the rest masses of the colliding objects change. (The germane part of Rindler’s treatment is reproduced at the end of this chapter as Addendum #1.) This turns out to be crucial to the prediction of Mach effects.

Now, Luchak’s relativistic Newtonian gravity equation looks very much like a standard classical field equation where the d’Alembertian operator (which involves taking spatial and temporal rates of change of a field quantity) acting on a field is equal to its sources. That is, it looks like a classical wave equation for the field with sources. It’s the time dependent term in  $q$  that messes this up because  $q$  is not  $\mathbf{F}$ .  $q$ , however, by definition, is the rate at which the field does work on sources, that is, the rate at which the energy of the sources changes due to the action of the field. So the term in  $q$  turns out to be the rate of change of the rate of change of the energy in a volume due to the action of the field on its sources. That is, it is the second time-derivative of the energy density.

This, the second time-derivative (of the field), is the correct form for the time-dependent term in the d’Alembertian of a field. The problem here is that the energy density isn’t the right thing to be acted upon by the second time-derivative if the equation is to be a classical wave equation. It should be the field itself, or a potential of the field that is acted on by the second time-derivative.

The interesting aspect of this equation is the ambiguity of whether the time-dependent term should be treated as a field quantity, and left on the left hand side of the equation, or if it can be transferred to the right hand side and treated as a source of the field. Mathematically, where the time-dependent term appears is a matter of choice, for subtracting a term from both sides of an equation leaves the equation as valid as the pre-subtraction equation.

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<sup>2</sup> More formalism:  $\Gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$